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Multibaryons in the collective coordinate approach to the SU(3) Skyrme model

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Abstract

We obtain the rotational spectrum of strange multibaryon states by performing the SU(3) collective coordinate quantization of the static multi-Skyrmions. These background configurations are given in terms of rational maps, which are very good approximations and share the same symmetries as the exact solutions. Thus, the allowed quantum numbers in the spectra and the structure of the collective Hamiltonians we obtain are also valid in the exact case. We find that the predicted spectra are in overall agreement with those corresponding to the alternative bound state soliton model.

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I. INTRODUCTION

In the Skyrme model [1] and its generalizations, baryons arise as topological excitations of a non-linear chiral Lagrangian written in terms of meson fields. These type of models have been quite successful in describing the properties of single baryons such as the nucleon and the strange hyperons (see e.g., Refs. [2,3]). This has lead people to investigate the lowest energy Skyrmion configurations with topological number greater than one, which are of inherent interest as examples of three dimensional solitonic structures and may also be relevant for nuclear physics. These studies were already started by Skyrme in his pioneer papers at the beginning of the sixties. However, it was only in 1987 that the minimum energy $B = 2$ Skyrmion was correctly identified [4]. Some time later the authors of Ref. [5] found the solutions with $B = 3, 4$ and 5 by numerical relaxation calculations. Finally, a few years ago [6], after some demanding numerical work, the global minimum energy configurations with topological number up to $B = 9$ were constructed. One particularly interesting aspect of all these multi-Skyrmion fields is that they are very symmetric. While for $B = 2$ the symmetry group corresponds to that of a torus, for $B = 3, 4, 7, 9$ they possess the symmetries of the platonic polihedra T_d, O_h, I_h and T_d, respectively, and for $B = 5, 6, 8$ the dihedral symmetries D_{2d}, D_{4d}, D_{6d}, respectively. It should be stressed that, in spite of this, the multi-Skyrmion fields are very complicated functions of the space coordinates which are only known numerically. Fortunately, rather simple and accurate approximations to these configurations have been found [7]. They are based on some *Ansätze* which are written in terms of rational maps and take advantage of the similarities between multi-Skyrmion fields and Bogomol'nyi-Prasad-Sommerfield (BPS) monopoles. These developments triggered several investigations concerning the properties of the multi-Skyrmions (such as e.g., vibrational excitations [8]) as well as their application to baryonic systems containing strangeness [9,10] and heavier flavors [11]. The extension to flavored multibaryons is also motivated by the advent of heavy ion colliders with the possibility of producing strange [12] and even charmed [13] multibaryonic states with rather low baryon number in the laboratory. To describe the strange multibaryons one has to extend the model to SU(3) flavor space. The classical background configurations are simply obtained by embedding the SU(2) static multi-Skyrmions in the isospin subgroup of SU(3). In order to obtain the spectrum with states of well defined spin and isospin quantum numbers, as well as their splittings, we have to perform the quantization of this system. However, the presence of the rather important symmetry breaking terms associated with the mass of the strange quark makes the quantization process not completely trivial. In fact, two alternative methods have been suggested in the literature. One is known as the bound state approach [14] (BSA) in which strange baryons are described as SU(2) rotating soliton-kaon bound systems. The other scheme assumes that the strange degrees of freedom can still be treated as rotational modes but the corresponding collective Hamiltonian is to be diagonalized exactly [15]. This method is usually called the rigid rotator approach (RRA). In two recent articles [9,10] SU(3) multi-Skyrmions have been investigated following the BSA. In this work we complement such investigations by considering these configurations within the framework of the RRA.

This paper is organized as follows. In Sec. II we provide a brief description of the model and obtain the collective Hamiltonian for the different baryon numbers. In Sec. III we focus on the determination of the multibaryon quantum numbers and wavefunctions. In Sec. IV

we present the numerical results and in Sec. V our conclusions. Finally, in the Appendix we give the explicit form of the collective Hamiltonians for $3 \leq B \leq 9$.

II. THE MODEL

We start with the effective action of the SU(3) Skyrme model supplemented with an appropriate symmetry breaking term [3]. Expressed in terms of the SU(3)-valued chiral field $U(x)$ it reads

$$\Gamma = \int d^4x \, \left\{ \frac{f_\pi^2}{4} \mathrm{Tr}\left[\partial_\mu U \partial^\mu U^\dagger \right] + \frac{1}{32e^2} \mathrm{Tr}\left[[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2 \right] \right\} + \Gamma_{WZ} + \Gamma_{SB} \ , \tag{1}$$

where f_π is the pion decay constant ($= 93$ MeV empirically) and e is the so-called Skyrme parameter. In Eq. (1), the symmetry breaking term Γ_{SB} accounts for the different masses and decay constants of the pion and kaon fields while Γ_{WZ} is the usual Wess-Zumino action. Their explicit forms are

$$\begin{aligned} \Gamma_{SB} = \int d^4x \Bigg\{ & \frac{f_\pi^2 m_\pi^2 + 2 f_K^2 m_K^2}{12} \mathrm{Tr}\left[U + U^\dagger - 2 \right] + \frac{f_\pi^2 m_\pi^2 - f_K^2 m_K^2}{6} \mathrm{Tr}\left[\sqrt{3} \lambda^8 \left(U + U^\dagger \right) \right] \\ & + \frac{f_K^2 - f_\pi^2}{12} \mathrm{Tr}\left[\left(1 - \sqrt{3}\lambda^8 \right) \left(U \partial_\mu U^\dagger \partial^\mu U + U^\dagger \partial_\mu U \partial^\mu U^\dagger \right) \right] \Bigg\} \ , \end{aligned} \tag{2}$$

$$\Gamma_{WZ} \ = \ -i \frac{N_c}{240\pi^2} \int \ d^5x \ \varepsilon^{\mu\nu\alpha\beta\gamma} \ \mathrm{Tr}\left[U^\dagger (\partial_\mu U) U^\dagger (\partial_\nu U) U^\dagger (\partial_\alpha U) U^\dagger (\partial_\beta U) U^\dagger (\partial_\gamma U) \right] \ , \tag{3}$$

where λ^8 is the eighth Gell-Mann matrix, N_c the number of colors, m_π and m_K are the pion and kaon masses, respectively, and f_K is the kaon decay constant.

We proceed by introducing the following *Ansatz* for the time dependent chiral field

$$U(\vec{\boldsymbol{r}}, t) = \mathcal{A}(t) \, \begin{pmatrix} \exp\left[i \vec{\boldsymbol{\tau}} \cdot \vec{\boldsymbol{\pi}}(\mathcal{R}^{-1}(t) \vec{\boldsymbol{r}}) \right] & 0 \\ 0 & 1 \end{pmatrix} \, \mathcal{A}^\dagger(t) \ , \tag{4}$$

where the embedded SU(2) background configuration is rigidly rotated both in SU(3) flavor space and real space, the collective coordinates given by $\mathcal{A}(t) \in$ SU(3) and $\mathcal{R}(t) \in$ SO(3), respectively. Substituting $U(\vec{\boldsymbol{r}}, t)$ given by Eq. (4) into the effective action yields a Lagrangian of the general form

$$L = -M_{sol} + L_{coll} \ , \tag{5}$$

where M_{sol} is the static SU(2) soliton mass and L_{coll} is the collective Lagrangian, whose general expression will be given below. Following the usual steps in the RRA, we first find the soliton background configuration by minimizing M_{sol}. For this purpose we introduce the rational map *Ansätze* [7] for the pion field

$$\vec{\boldsymbol{\pi}}(\vec{\boldsymbol{r}}) = F(r) \ \hat{\boldsymbol{n}} \ . \tag{6}$$

Here, $F(r)$ is the multi-Skyrmion profile which depends on the radial coordinate only and $\hat{\boldsymbol{n}}$ is a unit vector given by

$$\hat{\boldsymbol{n}} = \frac{1}{1+|R|^2}\left[2\,\Re(R)\;\hat{\boldsymbol{\imath}} + 2\,\Im(R)\;\hat{\boldsymbol{\jmath}} + (1-|R|^2)\;\hat{\boldsymbol{k}}\right]\;, \tag{7}$$

with $R = R(z)$ the rational map corresponding to a certain winding number B which is identified with the baryon number. The complex variable z is related to the usual two spherical coordinates (θ, ϕ) via stereographic projection, namely, $z = \tan(\theta/2)\exp(i\phi)$. For example, the map corresponding to the $B = 1$ hedgehog *Ansatz* is the identity map $R = z$. The explicit form of the rational maps corresponding to the other baryon numbers $B \leq 9$ and the resulting expression for the soliton mass M_{sol} can be found in Refs. [7,10]. The radial profile function $F(r)$ is determined by minimizing the classical soliton energy M_{sol}. Details of this procedure as well as plots of these profiles for different baryon numbers are given in Ref. [7].

The collective Lagrangian written in terms of the collective degrees of freedom and the corresponding angular velocities Ω_a, ω_α defined by [1]

$$\left(\mathcal{R}^{-1}\dot{\mathcal{R}}\right)_{ab} = \epsilon_{abc}\Omega_c\;, \tag{8}$$

$$\mathcal{A}^{-1}\dot{\mathcal{A}} = \frac{i}{2}\;\lambda_\alpha\;\omega_\alpha\;, \tag{9}$$

takes the general form

$$\begin{aligned} L_{coll} = \frac{1}{2}\sum_{a,b}\left(\Theta^J_{ab}\;\Omega_a\Omega_b + \Theta^N_{ab}\;\omega_a\omega_b + 2\;\Theta^M_{ab}\;\Omega_a\omega_b\right) + \frac{1}{2}\Theta^S\sum_k \omega_k^2 - \frac{N_c B}{2\sqrt{3}}\;\omega_8 \\ -\frac{1}{2}G_{SB}\left(1 - D_{88}\right)\;, \end{aligned} \tag{10}$$

with $D_{88} = \frac{1}{2}\;\mathrm{Tr}\left[\lambda_8\mathcal{A}\lambda_8\mathcal{A}^\dagger\right]$. The moment of inertia in the strangeness direction Θ^S is

$$\Theta^S = \int d^3r\;\frac{1-c}{2}\left[f_K^2 + \frac{1}{4e^2}\left(F'^2 + 2B\frac{s^2}{r^2}\right)\right]\;, \tag{11}$$

where we have introduced the short hand notation $s = \sin F$, $c = \cos F$. The spin Θ^J_{ab}, isospin Θ^N_{ab} and mixed moments of inertia Θ^M_{ab} are

$$\Theta^J_{ab} = \int d^3r\;s^2\;r^2\;\left[\left(f_\pi^2 + \frac{F'^2}{e^2}\right) + \frac{1}{2}\;\frac{s^2}{e^2}\;\nabla_c\hat{\boldsymbol{n}}\cdot\nabla_c\hat{\boldsymbol{n}}\right]\;\nabla_a\hat{\boldsymbol{n}}\cdot\nabla_b\hat{\boldsymbol{n}}\;, \tag{12}$$

$$\Theta^N_{ab} = \int d^3r\;s^2\;\left[\left(f_\pi^2 + \frac{F'^2}{e^2}\right)(\delta_{ab} - \hat{n}_a\hat{n}_b)\; + \frac{s^2}{e^2}\;(\delta_{ab} - 2\hat{n}_a\hat{n}_b)\;\nabla_c\hat{\boldsymbol{n}}\cdot\nabla_c\hat{\boldsymbol{n}}\right]\;, \tag{13}$$

$$\Theta^M_{ab} = -\int d^3r\;s^2\;r\;\left[\left(f_\pi^2 + \frac{F'^2}{e^2}\right) + \frac{1}{2}\;\frac{s^2}{e^2}\;\nabla_c\hat{\boldsymbol{n}}\cdot\nabla_c\hat{\boldsymbol{n}}\right]\;\nabla_a\hat{n}_b\;\;. \tag{14}$$

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[1] Here and in the following the spin/isospin indices a, b, c run over $\{1, ..., 3\}$, the flavor index α over $\{1, ..., 8\}$ and the $k \in \{4, ..., 7\}$ index corresponds to excitations into strangeness directions.

For the rational maps we are interested in all these moments of inertia are diagonal [10]. This is a direct consequence of the symmetries of these *Ansätze*. Finally, the symmetry breaking parameter G_{SB} is

$$G_{SB} = \frac{2}{3}(f_K^2 - f_\pi^2)\int d^3r \left(F'^2 + 2B\frac{s^2}{r^2}\right) c + \frac{4}{3}(f_K^2 m_K^2 - f_\pi^2 m_\pi^2)\int d^3r \ (1-c) \ . \tag{15}$$

Given L_{coll}, the spin and flavor canonical momentum operators $\hat{J}_a$ and $\hat{F}_\alpha$ are defined in the usual way

$$\hat{J}_a = \frac{\partial L_{coll}}{\partial \Omega_a} \qquad ; \qquad \hat{F}_\alpha = \frac{\partial L_{coll}}{\partial \omega_\alpha} \ . \tag{16}$$

The collective Hamiltonian is conventionally obtained as the Legendre transformation $H_{coll} = J_a\Omega_a + F_\alpha\omega_\alpha - L_{coll}$, resulting in

$$H_{coll} = K^S\left[C_2(SU(3)) - \frac{3}{4}B^2 - \hat{N}^2 + \gamma(1 - D_{88})\right] + H_B^{JN} \ . \tag{17}$$

Here, $C_2(SU(3)) = \sum_\alpha \hat{F}_\alpha^2$ stands for the quadratic SU(3) Casimir operator, $\hat{N}_a \equiv \hat{F}_a$ is the isospin operator in the soliton frame, $\gamma = \Theta^S G_{SB}$ is the dimensionless flavor symmetry breaking parameter and $K^S = 1/(2\Theta^S)$. In order to obtain Eq. (17) we have used $N_c = 3$ and the constraint $F_8 = -\frac{\sqrt{3}}{2}B$. Finally, the detailed form of the spin-isospin collective Hamiltonians H_B^{JN} depends on the soliton symmetry group. The method to derive them is very similar to the one described in Sec. III of Ref. [10]. For $B = 1, 2$ the corresponding groups are the continuous groups $O(3)$ and $D_{\infty h}$, respectively. In those cases there are some relations between the spin and isospin operators which lead to the well known expressions for the spin-isospin collective Hamiltonians

$$H_{B=1}^{JN} = \frac{1}{2\Theta^J}\hat{J}^2 \ , \tag{18}$$

$$H_{B=2}^{JN} = \frac{1}{2\Theta_1^J}\left(\hat{J}^2 - \hat{J}_3^2\right) + \frac{1}{2\Theta_1^N}\left(\hat{N}^2 - \hat{N}_3^2\right) + \frac{1}{2\Theta_3^N}\ \hat{N}_3^2 \ . \tag{19}$$

On the other hand, for $B \geq 3$ the symmetry groups are finite [6]. Thus, the general form of the spin-isospin collective Hamiltonian is

$$H_{B\geq 3}^{JN} = \sum_a \left(K_a^J\ \hat{J}_a^2 + K_a^N\ \hat{N}_a^2 - 2K_a^M\ \hat{J}_a\ \hat{N}_a\right) \ , \tag{20}$$

where

$$K_a^J = \frac{1}{2}\frac{\Theta_a^N}{\Delta_a} \ , \quad K_a^N = \frac{1}{2}\frac{\Theta_a^J}{\Delta_a} \ , \quad K_a^M = \frac{1}{2}\frac{\Theta_a^M}{\Delta_a} \ . \tag{21}$$

and $\Delta_a \equiv \Theta_a^J\Theta_a^N - (\Theta_a^M)^2$. The explicit form for each baryon number can be found in the Appendix.

III. QUANTUM NUMBERS AND COLLECTIVE WAVE FUNCTIONS

In order to calculate the rotational corrections to the multi-Skyrmion masses we have to find the corresponding wave functions. Following the Yabu-Ando procedure [15], they should diagonalize the flavor symmetry breaking term in the collective Hamiltonian. At the same time they should satisfy the constraints imposed by the symmetries of the classical soliton configuration. Thus, the general form of such eigenfunctions will be

$$|BJJ_z, YII_z, N\rangle = \sum_{J_3 N_3} \alpha^{JN}_{J_3 N_3} \, D^J_{J_z J_3} \, \Psi_{(Y,I,I_z),(B,N,N_3)} \ , \tag{22}$$

Here, $D^J_{J_z J_3}$ is the usual SU(2) Wigner function and $\Psi_{(Y,I,I_z),(B,N,N_3)}$ is a function depending on the 8 Euler angles that parametrize the SU(3) manifold. To obtain $\Psi_{(Y,I,I_z),(B,N,N_3)}$ we should solve the eigenvalue equation

$$K^S \left[h + \gamma(1 - D_{88})\right] \ \Psi = \epsilon \ \Psi \ , \tag{23}$$

where $h = C_2(SU(3)) - \frac{3}{4}B^2 - N(N+1)$. The coefficients $\alpha^{JN}_{J_3 N_3}$ are determined in such a way that the full wavefunction transforms as some particular one-dimensional irreducible representation (irrep) of the soliton symmetry group G. This will be discussed in some detail below.

To solve Eq. (23) we expand $\Psi_{(Y,I,I_z),(B,N,N_3)}$ in a basis of SU(3) Wigner functions $D^{(p,q)}_{(Y,I,I_z),(B,N,N_3)}$, where (p,q) are the labels used to identify the SU(3) irrep. Namely,

$$\Psi_{(Y,I,I_z),(B,N,N_3)} = \sum_{(p,q)} \beta_{(p,q)} \ \sqrt{d_{(p,q)}} \ D^{(p,q)}_{(Y,I,I_z),(B,N,N_3)} \ , \tag{24}$$

where $d_{(p,q)} = (p+1)(q+1)(p+q+2)/2$ is the dimension of the irrep. In such basis h is diagonal and the matrix elements of the symmetry breaking term can be expressed as a product of two SU(3) Clebsch-Gordan coefficients. To determine, for a given value of B, the allowed values of the Y, I and N quantum numbers as well as which SU(3) irrep should be included in the basis we proceed as follows. As already seen, the value of the right hypercharge $Y_R \equiv -2F_8/\sqrt{3}$ is fixed by the constraint $Y_R = B$. Thus, any SU(3) irrep that appears in the expansion, Eq.(24), should have a maximum value of hypercharge equal or larger than B. Thus, the possible values of (p,q) should satisfy

$$\frac{p+2q}{3} = B + m \ , \tag{25}$$

with p and q non-negative integer numbers and $m = 0, 1, 2, ...$ The irreps corresponding to $m = 0$ are the so-called minimal irreps that we will denote (p_0, q_0). It is possible to show [16] that the matrix element of h in any state that belongs to a minimal irrep is $< h >_0 = 3B/2$. To determine the relevant values of Y, I and N it is enough to consider such irreps. Although for non-vanishing strangeness S other values of the quantum numbers could be allowed, they will be of no interest to us. In fact, it is not difficult to show that for the first state with "non-minimal quantum numbers" the matrix element of h is more than twice $< h >_0$. Therefore, such state is expected to appear as a highly excited state in

the spectrum. Since the minimal irreps have maximum right hypercharge $Y_R = B$ it is clear that corresponding possible values of the body-fixed isospin N are $N = p_0/2$. On the other hand, those of the hypercharge Y are

$$-\frac{2p_0 + q_0}{3} \leq Y \leq \frac{p_0 + 2q_0}{3} \ . \tag{26}$$

Finally, given a value of Y that satisfies this relation, the allowed values of the isospin I are

$$\left|\frac{Y}{2} + \frac{p_0 - q_0}{3}\right| \leq I \leq \frac{p_0 + q_0}{2} - \frac{1}{2}\left|Y - \frac{p_0 - q_0}{3}\right| \ . \tag{27}$$

In Table I we list, for each baryon number $3 \leq B \leq 9$, the minimal SU(3) irrep which lead to states with $N < 3$ together with the allowed values of isospin for some values of strangeness. Given a set of possible (B, I, Y, N) quantum numbers one should find all the SU(3) irreps with $m > 0$ that enter in the expansion, Eq. (24). This is done by selecting from all the irreps which satisfy Eq. (25) those that contain a state with this same set of quantum numbers. This leads to different towers of SU(3) irreps for each set of quantum numbers. Once this is done, it is a simple task to transform Eq. (23) into an ordinary linear eigenvalue problem whose solution provides the energy eigenvalues ϵ and the coefficients $\beta_{(p,q)}$. Of course, to do that one should work with a basis of finite size. Since we are interested only in the few lowest eigenvalues the minimum size is fixed by the condition that those eigenvalues remain unchanged under a further increase of such size.

Having determined $\Psi_{(Y,I,I_z),(B,N,N_3)}$ and the corresponding possible quantum numbers we have still to obtain the coefficients $\alpha^{JN}_{J_3N_3}$ of Eq. (22) and the allowed values of J. For this purpose, only the spin J and isospin N are relevant. Thus, the situation is very similar to that of the $S = 0$ case discussed in Sec. IV of Ref. [10]. As already mentioned the full wave function should transform as a one dimensional irrep of the multisoliton symmetry group G. For the configurations we are dealing with we have that, except for the $B = 5$ and $B = 6$ cases, such one dimensional irrep is the trivial irrep of the corresponding symmetry groups. For $B = 5$, Γ is the A_2 irrep of D_{2d}, while for $B = 6$ the wave functions should transform as the A_2 irrep of D_{4d}. Using standard group theoretical arguments [17] we know that the product representation $J \times N$ of SU(2) is in general a reducible representation of G. The projector operator into the one dimensional irrep Γ is

$$P_\Gamma = \frac{1}{|G|} \sum_{g \in G} \chi^*_\Gamma(g)\, \rho(g) \ , \tag{28}$$

where $|G|$ is the rank of the group, $\chi_\Gamma(g)$ the character of operation g, and $\rho(g)$ the representation of g in $J \times N$

$$\rho(g) = D^J(g) \times D^N(D_g) \ . \tag{29}$$

where D_g is the isospin operation associated with the space operation g. The eigenvalues of P_Γ can either vanish or be equal to one. The eigenvectors corresponding to each non-vanishing eigenvalue provide precisely the coefficients $\alpha^{JN}_{J_3N_3}$ of Eq. (22), and there are as many wave functions as non-zero eigenvalues. If all eigenvalues vanish there is no collective state with the given J, N. If there is only one, the wavefunction is an eigenfunction of the collective Hamiltonian. In case there would be more than one, we choose those combinations which diagonalize the parity operator.

IV. NUMERICAL RESULTS

To calculate the multibaryon spectra we use the following set of values for the parameters appearing in the effective action, Eqs. (1-3). We fix f_π, m_π and m_K to their empirical values and take $e = 4.1$ and $f_K/f_\pi = 1.29$. This set of parameters leads to a single baryon excitation spectrum which is in very good agreement with the one observed for the octet and decuplet baryons. As well known, however, the use of the empirical value for f_π implies a $B = 1$ Skyrmion mass of around 1.7 GeV. Consequently, the absolute values of the calculated masses come out to be too large. This problem is nowadays known to be solved by the inclusion of Casimir effects [18,19]. We will come back to this issue below. With these values we can calculate M_{sol} and the different quantities that appear in the expression of L_{coll} given by Eq. (10). The results for the different baryon numbers up to $B = 9$ are tabulated in Tables II and III. From Table II we observe that although M_{sol}/B tends, on average, to decrease as a function of B it always lies above 1.5 GeV. This clearly indicates that Casimir effects will be also important to determine the absolute masses of the configurations with $B > 1$. In any case, as in previous works where f_π was adjusted to reproduce the empirical nucleon mass, we observe some deviation from a smooth behaviour. Also listed in Table II are the strange inertia parameter K^S and the symmetry breaking parameter γ. We see that, roughly, K^S decreases as $1/B$ while γ increases as B^2. As we will see this has important consequences on the amount of configuration mixing as a function of the baryon number. In Table III we list the spin, isospin and mixing inertia parameters for the different values of B. We find that the values we obtained behave, as a function of B, as those of Ref. [10]. In fact this is to be expected since, as explained in that reference, such behaviour as well as the number of independent components depends only on general properties of the *Ansätze*.

Given the values of the inertia and symmetry breaking parameters we can proceed to calculate the matrix elements of the rotational Hamiltonian. For this purpose we have to find the solutions of the eigenvalue equation Eq. (23). As explained in the previous section this amounts to determine the coefficients $\beta_{(p,q)}$ appearing in Eq. (24). We have done this calculation for the different sets of allowed quantum numbers. It interesting to note that the amount of configuration mixing increases with B. This can be clearly observed in Fig. 1 where we display the decomposition of the lowest energy states with strangeness $S = 0$ (full line) and $S = -B$ (dashed line) for $B = 3$ and $B = 9$. In this figure the symbol i labels the different (p, q) irrep that appear in each decomposition. Of course, $i = 0$ indicates the corresponding minimal irrep. We see that while for $B = 3$ about 80% of the wavefunction corresponds to the minimal irrep, for $B = 9$ such irrep represents less that 30% with the rest of strength distributed in almost 10 irreps. This kind of behaviour can be simply understood using second order perturbation theory. Within that approximation $\beta_{i=1}$, that is the coefficient of the first non-minimal irrep, will be proportional to $\gamma/(< h >_1 - < h >_0)$. It is not hard to show that, for the ground states with $S = 0$, one has $(< h >_1 - < h >_0) \propto B$. Since we have seen that $\gamma \propto B^2$ we obtain that $\beta_{i=1}$ should increase roughly as B. Similar arguments can be used for the case $S = -B$. This explains why the configuration mixing is quite independent of the value of strangeness as it can be seen in Fig. 1 by comparing the solid lines with the dashed ones. From the numerical point of view the increase of configuration mixing implies that as larger values of B are considered one has to increase the size of the basis in which the eigenfunction is expanded in order to obtain convergence.

In all the cases of interest we found that no more than 15 to 20 configurations were needed.

The resulting multibaryon spectra are summarized in Tables IV and V. In Table IV we report the rotational corrections to the masses of the $S = 0$ states. They are given as excitation energies taken with respect to the corresponding lowest energy state whose absolute rotational energy is indicated in brackets. It is important to mention that for the $B = 1$ systems it was shown that this rather large absolute value is almost completely cancelled by the Casimir corrections due to kaon loops [19]. Since similar cancellations are expected to happen for $B > 1$, the excitation energies result to be the most meaningful quantities to look at. We observe that the predicted spectra are in agreement with the ones obtained in the alternative bound state approach [10], except for a few changes in the ordering of the states in the case of $S = -B$ and $B = 5, 8, 9$. From the numbers presented in Tables II, IV and V it is apparent that there is a clear separation of three different energy scales. There is a 1 GeV scale related to the classical masses (per baryon number) and the eigenvalues of Eq. (23) for $S = 0$ states, there is another scale of about 300 MeV for the excitation of one unit of strangeness, and finally a 10-100 MeV scale related to spin-isospin excitations. This last energy scale is evident in Table IV while it appears as a small correction in Table V. In this way we recover the three leading order contributions in the N_c expansion N_c, N_c^0 and N_c^{-1}, which are more explicitly separated in the BSA.

V. CONCLUSIONS

In this work we have studied the multibaryon spectra for baryon number $3 \leq B \leq 9$ and strangeness values $S = 0, -1, -B$ within the SU(3) collective coordinate approach to the three flavor Skyrme model. To describe the classical background solutions we have used *Ansätze* based on rational maps [7], which provide very good approximations and also share the same symmetries as the exact solutions. The symmetry structure is responsible for the spin and isospin assignments to the spectrum states. Therefore, the collective Hamiltonians and wave functions we obtain are of general validity, while the mass splittings depend on the particular values of the moments of inertia and of the symmetry breaking parameter.

We have found that, in general, the ordering of the different spin/isospin states corresponding to a given baryon number as well as the energy separation between those states obtained by using the present approach are very similar to the results of the alternative bound state treatment of the SU(3) Skyrme model. This fact together with the observation that in the collective approach the relative strength of the flavor symmetry breaking term increases with increasing baryon number (cf., Fig. 1) seems to indicate that both approaches tend to coincide as B grows. In this sense we can conclude that our finding that the increase of one unit of strangeness implies a cost in energy of about 300 MeV rather independently of $B \geq 3$ appears to be a rather general prediction of the SU(3) Skyrme model.

Finally, note that in the present calculation we have set the meson decay constants to their empirical values. Consequently, all the resulting absolute masses are too large. For example, we find values of M_{sol}/B of about 1.60 GeV and $S = 0$ ground state rotational corrections of about 0.8 GeV. These values are expected to be largely compensated by the pion and kaon contributions to the Casimir energies, respectively. In fact, this has been recently shown to happen in the $B = 1$ sector of the model [19]. Unfortunately, for $B > 1$

the difficulties associated with the treatment of the fluctuations around non-spherically symmetric soliton backgrounds have prevented so far the explicit evaluation of the Casimir effect even in the SU(2) case.

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APPENDIX:

In this Appendix we give the explicit expressions of the spin-isospin collective Hamiltonian for $B \geq 3$. The form of these expressions depends only on the symmetries of the soliton configurations. The method to derive them is very similar to the one described in Sec. III of Ref. [10]. In fact, the following expressions can be easily obtained from the ones given in that reference by setting the corresponding hyperfine splitting constants to zero.

$$H^{JN}_{B=3} = H^{JN}_{B=9} = K^J\ \hat{J}^2 + K^N\ \hat{N}^2 - 2K^M\ \vec{\hat{N}} \cdot \vec{\hat{J}}\ , \tag{A1}$$

$$H^{JN}_{B=4} = K^J\ \hat{J}^2 + K^N_1\ \hat{N}^2 + (K^N_3 - K^N_1)\ \hat{N}^2_3\ , \tag{A2}$$

$$\begin{aligned} H^{JN}_{B=5} = {} & K^J_1\ (\hat{J}^2 - \hat{J}^2_3) + K^N_1\ (\hat{N}^2 - \hat{N}^2_3) - 2K^M_1\ (\vec{\hat{N}} \cdot \vec{\hat{J}} - \hat{N}_3\ \hat{J}_3) \\ & + K^J_3\ \hat{J}^2_3 + K^N_3\ \hat{N}^2_3 - 2K^M_3\ \hat{N}_3\ \hat{J}_3\ , \end{aligned} \tag{A3}$$

$$H^{JN}_{B=6} = H^{JN}_{B=8} = K^J_1\ \hat{J}^2 + K^N_1\ \hat{N}^2 + (K^J_3 - K^J_1)\ \hat{J}^2_3 + (K^N_3 - K^N_1)\ \hat{N}^2_3 - 2K^M_3\ \hat{N}_3\ \hat{J}_3\ , \tag{A4}$$

$$H^{JN}_{B=7} = K^J\ \hat{J}^2 + K^N\ \hat{N}^2\ . \tag{A5}$$

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TABLES

TABLE I. Minimal SU(3) irreps and allowed values of N and I for states with some selected values of strangeness for $B = 3 - 9$. Only states with $N < 3$ are listed.

B	Minimal SU(3) irrep	N	Allowed values of I		
			$S = 0$	$S = -1$	$S = -B$
3	$\overline{\mathbf{35}}$	1/2	1/2	0, 1	1,2
	$\mathbf{64}$	3/2	3/2	1, 2	0,1,2,3
	$\mathbf{81}$	5/2	5/2	2, 3	1,2,3
4	$\overline{\mathbf{28}}$	0	0	1/2	2
	$\overline{\mathbf{81}}$	1	1	1/2, 3/2	1,2,3
	$\mathbf{125}$	2	2	3/2, 5/2	0,1,2,3,4
5	$\overline{\mathbf{80}}$	1/2	1/2	0, 1	2,3
	$\overline{\mathbf{154}}$	3/2	3/2	1, 2	1,2,3,4
	$\mathbf{216}$	5/2	5/2	2, 3	0,1,2,3,4,5
6	$\overline{\mathbf{55}}$	0	0	1/2	3
	$\overline{\mathbf{162}}$	1	1	1/2, 3/2	2,3,4
	$\overline{\mathbf{260}}$	2	2	3/2, 5/2	1,2,3,4,5
7	$\overline{\mathbf{143}}$	1/2	1/2	0, 1	3,4
	$\overline{\mathbf{280}}$	3/2	3/2	1, 2	2,3,4,5
	$\overline{\mathbf{405}}$	5/2	5/2	2, 3	1,2,3,4,5,6
8	$\overline{\mathbf{91}}$	0	0	1/2	4
	$\overline{\mathbf{270}}$	1	1	1/2, 3/2	3,4,5
	$\overline{\mathbf{440}}$	2	2	3/2, 5/2	2,3,4,5,6
9	$\overline{\mathbf{224}}$	1/2	1/2	0, 1	4,5
	$\overline{\mathbf{442}}$	3/2	3/2	1, 2	3,4,5,6
	$\overline{\mathbf{648}}$	5/2	5/2	2, 3	2,3,4,5,6,7

TABLE II. Soliton mass (per baryon unit), strangeness inertia parameter and symmetry breaking parameter for $B = 3 - 9$.

B	M_{sol}/B (GeV)	K^S (MeV)	γ
3	1.64	55.12	38.43
4	1.58	43.18	61.18
5	1.59	32.80	105.69
6	1.58	27.03	154.83
7	1.54	23.96	194.76
8	1.56	20.04	279.99
9	1.57	17.25	379.42

TABLE III. Spin, isospin and mixed inertia parameters for $B = 3 - 9$.

B	K^J (MeV)	K^N (MeV)	K^M (MeV)
3	11.28	37.99	7.19
4	6.29	28.94	0
		28.94	
		24.10	
5	3.77	20.74	-0.88
	3.77	20.74	-0.88
	4.27	24.71	-0.67
6	2.66	19.06	0
	2.66	19.06	0
	3.09	17.93	0.94
7	2.23	16.72	0
8	1.73	14.23	0
	1.73	14.23	0
	1.53	15.38	-0.47
9	1.29	13.03	-0.33

TABLE IV. Quantum numbers and rotational excitation energies for the $S = 0$ states. The excitation energies are taken with respect to that of the lowest energy state for each baryon number. The absolute rotational energies of those states are indicated in brackets.

B	J^P	I	N	E_{exc}(MeV)
3	$1/2^+$	1/2	1/2	(847)
	$5/2^-$	1/2	1/2	61
	$3/2^-$	3/2	3/2	110
4	0^+	0	0	(808)
	4^+	0	0	126
	0^+	2	2	180
5	$1/2^+$	1/2	1/2	(837)
	$3/2^+$	1/2	1/2	9
	$3/2^-$	1/2	1/2	11
6	1^+	0	0	(827)
	3^+	0	0	27
	0^+	1	1	34
7	$7/2^+$	1/2	1/2	(872)
	$3/2^+$	3/2	3/2	24
	$9/2^+$	3/2	3/2	71
8	0^+	0	0	(828)
	2^+	0	0	10
	1^+	1	1	32
9	$1/2^+$	1/2	1/2	(842)
	$5/2^-$	1/2	1/2	12
	$7/2^-$	1/2	1/2	18

TABLE V. Quantum numbers and rotational excitation energies (per unit of strangeness) for $S = -1$ and $S = -B$ states. The excitation energies (in MeV) are taken with respect to that of the $S = 0$ lowest energy state for each baryon number. The absolute rotational energies of those states are given in Table IV.

	$S=-1$				$S=-B$			
B	J^P	I	N	$E_{exc}/\lvert S\rvert$	J^P	I	N	$E_{exc}/\lvert S\rvert$
3	$1/2^+$	0	1/2	263.5	$1/2^+$	1	1/2	291.7
	$1/2^+$	1	1/2	304.0	$3/2^-$	0	3/2	292.9
	$5/2^-$	0	1/2	325.0	$5/2^+$	0	3/2	304.4
4	0^+	1/2	0	287.9	0^+	0	2	302.8
	4^+	1/2	0	413.7	0^+	2	0	308.5
	0^+	3/2	2	425.1	0^+	1	2	311.8
5	$1/2^+$	0	1/2	279.4	$1/2^+$	1	3/2	301.7
	$3/2^+$	0	1/2	288.1	$1/2^-$	1	3/2	303.6
	$3/2^-$	0	1/2	290.8	$3/2^-$	1	3/2	304.7
6	1^+	1/2	0	299.1	0^-	1	2	308.6
	0^+	1/2	1	313.5	1^-	1	2	309.5
	3^+	1/2	0	325.7	1^+	1	2	310.3
7	$7/2^+$	0	1/2	282.0	$3/2^+$	2	3/2	301.3
	$3/2^+$	1	3/2	298.7	$5/2^+$	1	5/2	302.8
	$7/2^+$	1	1/2	299.1	$7/2^+$	1	5/2	305.1
8	0^+	1/2	0	301.3	0^+	2	2	313.5
	2^+	1/2	0	311.7	1^+	1	3	314.8
	1^+	1/2	1	319.3	2^-	2	2	314.8
9	$1/2^+$	0	1/2	296.5	$1/2^-$	2	5/2	318.0
	$5/2^-$	0	1/2	308.1	$3/2^-$	2	5/2	318.3
	$1/2^+$	1	1/2	309.4	$5/2^+$	2	5/2	318.4

FIGURES

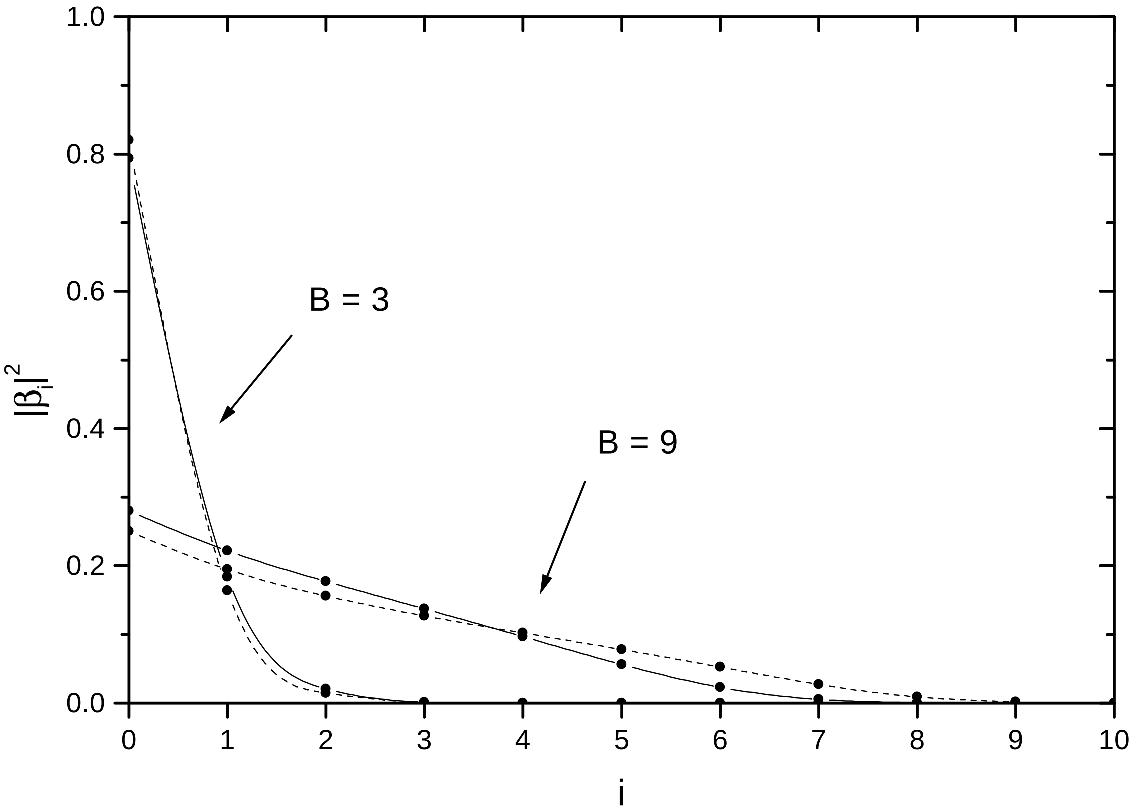

FIG. 1. Contribution of higher irreps to the lowest energy states with $S = 0$ (full line) and $S = -B$ (dashed line).